Section 6.2 – Law of COSINES

The two remaining cases for solving oblique triangles, SSS and SAS, cannot be solved using the Law of Sines because none of the ratios would be complete. In these two cases, the Law of Cosines must be used.

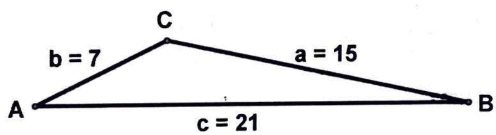
Law of Cosines

Standard Form – use with SAS	Alternative Form – use with SSS
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

** When using the alternative form, **
** YOU MUST FIND THE LARGEST ANGLE FIRST **

SSS Case

1) Find the three angles of the triangle in the figure below



$m\angle A =$ 25.66°

$m\angle B =$ 11.66°

$m\angle C =$ 142.68°

★ Largest ∠ must be C (opposite longest side)

$$\therefore \cos C = \frac{15^2+7^2-21^2}{2(15)(7)} = \frac{-167}{210}$$

$$\Rightarrow C = 142.68°$$

Now use Law of Sines!

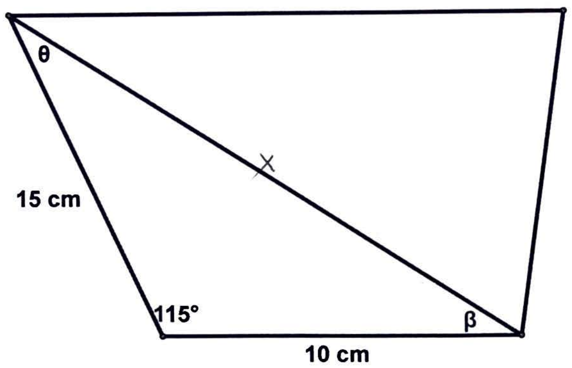
$$\frac{\sin A}{15} = \frac{\sin 142.68°}{21} \qquad \frac{\sin B}{7} = \frac{\sin 142.68°}{21}$$

$$A = 25.66° \qquad B = 11.66°$$

Section 6.2 – Law of COSINES

SAS Case

2) Find the length of the diagonal of the trapezoid, and the missing angles.



θ = 25.24°

β = 39.76°

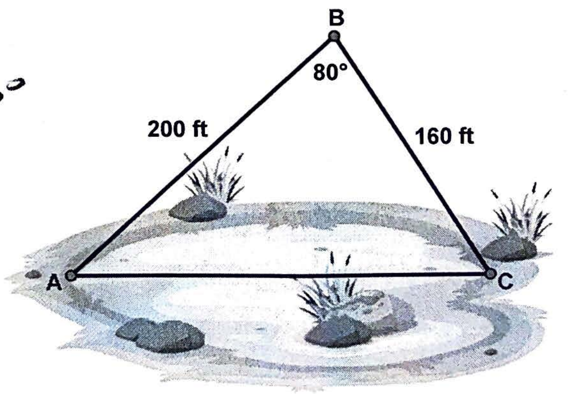
diagonal = 21.26

$x^2 = 15^2 + 10^2 - 2(15)(10)\cos 115°$

$x = 21.26$

$\frac{\sin\theta}{10} = \frac{\sin 115°}{21.26} = \frac{\sin\beta}{15}$

3) To approximate the length of a pond, you walk 200 ft from point A to point B, turn 80° and walk 1~~7~~0 ft. to point C. Approximate the length AC of the pond.
6

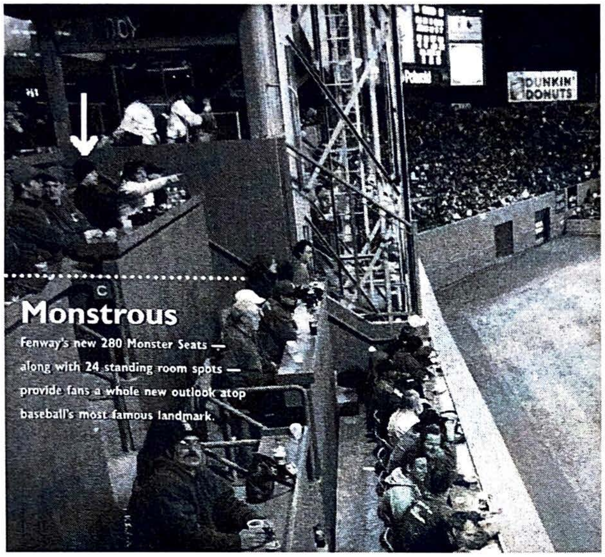


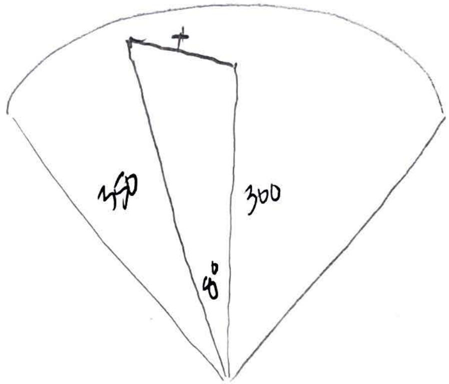
$AC^2 = 200^2 + 160^2 - 2(200)(160)\cos 80°$

AC = 233.42 ft

Section 6.2 – Law of COSINES

4) Mookie Betts is playing centerfield against the Yankees. He is approximately 300 ft. from the television camera that is behind home plate. Jacoby Ellsbury hits a fly ball that goes to the green monster 350 ft. away from the camera. The camera turns 8° to follow the play. Approximately how far does Betts have to run to the make the catch?





$$x = \sqrt{350^2 + 300^2 - 2(350)(300)\cos 8^\circ}$$

$$= 67.41 \text{ ft}$$

Section 6.2 – Law of COSINES

Heron's Area Formula

Heron's Area Formula

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

5) Find the area of a triangle having sides of length a = 75.4, b = 52, and c = 52.

$$s = \frac{75.4+52+52}{2} = 89.7$$

$$A = \sqrt{89.7(89.7-75.4)(89.7-52)^2}$$

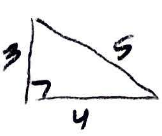
$$= 1350.22\ u^2$$

6) Find the area of a triangle with sides of length a = 3, b = 4, and c = 5

$$s = \frac{3+4+5}{2} = 6$$

$$A = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6\cdot 3\cdot 2} = \sqrt{36} = 6\ u^2$$

Is there a way to check your answer for this one?



$$A = \tfrac{1}{2}(4)(3) = 6\ u^2 \checkmark !$$

How do we know WHICH area formula to use?? (Recall: we learned another one with Law of Sines)

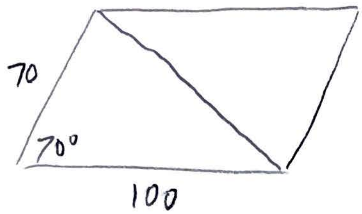
SSS → use Heron's

SAS → use $\frac{1}{2}bc\sin A$

right Δ → use $\frac{1}{2}bh$

anything else, you need to find more info using Law of Sines first!

7) A parking lot has the shape of a parallelogram. The lengths of the two adjacent sides are 70 meters and 100 meters. The angle between the two sides is 70°. What is the area of the parking lot?



Area of $\Delta = \frac{1}{2}(70)(100)\sin 70°$

$= 3288.92$

Area of ▱ = 2 area of Δ

$\boxed{= 6577.85 \text{ m}^2}$

Homework: p. 443 #1, 3, 6, 9, 23, 24, 31, 34, 39